WNE Linear Algebra
Resit Exam
Series B

28 February 2020

Please use separate sheets for different problems. Please answer all questions on a single sheet. Give reasons to your answers. Please provide the following data on each sheet

- **name, surname and your student number,**
- **number of your group,**
- **number of the corresponding problem and the series.**

Each problem is worth 10 marks and each question is worth 5 marks. To pass it necessary (but not sufficient) to score at least 50% out of 6 questions.

Problem 1.
Let $V = \text{lin}((1,1,5,2,8),(1,2,4,4,8),(3,13,5,26,24))$ be a subspace of $\mathbb{R}^5$.

a) find a basis of the subspace V and the dimension of V,
b) find a system of linear equations which set of solutions is equal to V.

Problem 2.
Let $V \subset \mathbb{R}^5$ be a subspace given by the homogeneous system of linear equations

$$\begin{cases} x_1 + 2x_2 + 10x_3 + 7x_4 + 9x_5 = 0 \\ x_1 + 3x_2 + 13x_3 + 8x_4 + 14x_5 = 0 \end{cases}$$

a) find a basis $\mathcal{A}$ of the subspace V and the dimension of V,
b) extend the basis $\mathcal{A}$ to a basis $\mathcal{B}$ of $\mathbb{R}^5$ and find coordinates of vector $w = (1,-2,1,-1,0) \in \mathbb{R}^5$ relative to the basis $\mathcal{B}$.

Problem 3.
Let

$$A = \begin{bmatrix} -4 & 3 \\ -6 & 5 \end{bmatrix}.$$

a) find a matrix $C \in M(2 \times 2; \mathbb{R})$ such that

$$C^{-1}AC = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix},$$

where $b < 0$,
b) compute A^{45}.

Problem 4.
Let

$$A = \begin{bmatrix} 2 & t & 1 \\ t & 2 & t \\ 1 & 2t & 2 \end{bmatrix}.$$

a) for which $t \in \mathbb{R}$ is the matrix $(A^\top)^4 A^3 (A^\top)^2$ invertible?
b) find the entry in the second row and the third column of the matrix A^{-1} for $t = 1$.

Problem 5.
Consider the following linear programming problem $-4x_2 - 3x_3 - x_4 \to \min$ in the standard form with constraints

$$\begin{cases} \quad\quad\quad x_2 + x_3 = 1 \\ -x_1 + 2x_3 + x_4 = 0 \\ x_1 - 2x_3 = 2 \end{cases} \quad \text{and } x_i \geqslant 0 \text{ for } i = 1, \ldots, 4$$

a) which of the sets $\mathcal{B}_1 = \{2,3,4\}$, $\mathcal{B}_2 = \{1,2,4\}$, $\mathcal{B}_3 = \{1,2,3\}$ are basic? Which basic set is basic feasible? Write the corresponding feasible solution.
b) solve the linear programming problem using simplex method.

Questions

Question 1.
Is matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ negative definite for all $a, b, c, d < 0$?

Question 2.
If $v = (1,0,1), w = (1,2,3) \in \mathbb{R}^3$, $V = \operatorname{lin}(v)$, is the image of vector $w \in \mathbb{R}^3$ under the (linear) orthogonal projection on the subspace $V^\perp \subset \mathbb{R}^3$ equal to

$$P_{V^\perp}(w) = (-1,2,1)?$$

Question 3.
If $A \in M(2 \times 2; \mathbb{R})$ and $A - A^\intercal = \mathbf{0}$, does it follow that $A^\intercal A = AA^\intercal$?

Question 4.
Is it possible that $A, B \in M(2 \times 2; \mathbb{R})$, $\det B = 0$ and $\det\big(2AB + B^2\big) \neq 0$?

Question 5.
Is it possible that $\mathcal{A}, \mathcal{B}$ are two different bases of $\mathbb{R}^2$ and

$$M(\mathrm{id})_{\mathcal{A}}^{\mathcal{B}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}?$$

Question 6.
Are the affine subspaces $E,\ H \subset \mathbb{R}^3$ given by

$$E = (1,1,2) + \operatorname{aff}((-1,0,1),(1,1,2),(3,2,3)),$$
$$H = (1,-1,0) + \operatorname{lin}((2,1,1)),$$

parallel?